Definition of the observer/identifier and the algorithms used

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■ **Abstract:**

The ability to estimate unknown system states and parameters is an essential requirement in control theory. Often system parameters are partially known and some of the states are inaccessible. This indicates that an Observer/Identifier is needed so that a control strategy can be applied effectively. An observer is a system whose state variables are the estimates of the state variables of the unknown system. Similarly, an Identifier is a system whose state variables are the estimates of the unknown system parameters. The ability to reconstruct the system states and parameters from the I/O data requires that all of the states must be observable. An on-line estimation and identification is usually desired. The success of an adaptive observer/identifier depends very much on the choice of the system model representing the unknown system. The choice of the order of the system model and whether the system model is time-variant or not, usually depends on a prior knowledge of the system dynamics and/or the study of the input-output data, such as frequency response, time domain response, etc. Once a system model is selected, an observer/identifier which usually adopts and extends an existing algorithm is developed.

- **Keywords:** *Observer/Identifier; System Parameters; System States; estimation algorithms;*

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■ المستخلص:

تُعد القدرة على تقدير حالات النظام ومعاملاته غير المعروفة من المتطلبات الجوهرية في نظرية التحكم. ففي العديد من التطبيقات، تكون بعض معاملات النظام معروفة جزئياً، بينما تكون بعض حالاته غير قابلة للقياس المباشر. وهو ما يستدعي تطوير مراقب/مُعرّف يمكن من خلاله تنفيذ استراتيجية تحكم فعّالة.

يُعرف المراقب على أنه نظام تكون متغيرات حالته بمثابة تقديرات لحالات النظام غير المعروفة، في حين يُستخدم المُعرّف لتقدير المعاملات غير المعروفة للنظام. ويتطلب تحقيق القدرة على إعادة بناء حالات النظام ومعاملاته انطلاقاً من بيانات الإدخال والإخراج وأن تكون جميع الحالات قابلة للمراقبة. وعادةً ما يُفضّل أن تتم عملية التقدير والتعريف بصورة آنية. يعتمد نجاح المراقب/المُعرّف التكيفي إلى حد كبير على اختيار نموذج مناسب للنظام المجهول. ويتوقف تحديد رتبة النموذج، وكذلك ما إذا كان النموذج متغيراً زمنياً أم لا، على المعرفة المسبقة بديناميكية النظام أو من خلال تحليل بيانات الإدخال/الإخراج، مثل استجابة التردد أو استجابة الزمن، إلخ. وبمجرد تحديد نموذج النظام المناسب، يتم تطوير مراقب/مُعرّف يستند في الغالب إلى خوارزمية قائمة يتم تعديلها أو توسيعها لتتناسب مع خصائص النظام قيد الدراسة.

● الكلمات المفتاحية : مراقب/مُعرّف؛ معاملات النظام؛ حالات النظام؛ خوارزميات التقدير؛ مرشح كالمان؛ طريقة المربعات الصغرى التكرارية.

■ INTRODUCTION

In order to apply control theory effectively to a particular system, the problem must be well poised both in parameter and state. Many times a system is not well known and the following problems arise:

1. The system general dynamics are known but its coefficients are uncertain and may also be a function of time.

2. Not all states are measurable, or it is not practical to measure them.

The above problems necessitate the need for an observer and/or identifier. An nth-order linear discrete time-variant single-input single-output model is considered

$$\begin{cases} x(k+1) = F(k)x(k) + G(k)u(k) \\ y(k) = r(k)x(k) \end{cases} \tag{1}$$

Where *x(k) is an nth-order state vector, F(k) is an n × n matrix, G(k) is an n × 1 vector, r(k) is a 1 × n vector, u(k)* is the single input, and y(k) is the single output.

The proposed problem is to devise an adaptive scheme which would

·estimate the state vector z(t), and

·identify the parameters (*F(k)*,$G(k)$, *r(k)*), given that nothing is known about the system except that it is of order n, linear, and time-invariant, and that the only available signals are *u(k)* and *y(k)*.

■ COMMON ALGORITHMS

Observers/identifiers represent an essential advance in modern control theory and are treated extensively in the literature. There are many unique approaches to observer/identifier design. In this paper we will go through a list of the most widely used methods.

·A well-known class of observers/identifiers is based on Kalman filter algorithm [1]. When the system parameters are not known, they are combined with the states in one vector to derive at the Extended Kalman filter. The augmentation of the states with the parameters leads to a nonlinear system model which in some sense has some undesirable characteristics.

·The second kind of observer/identifier is based on the stability criteria and in particular Liapunov stability function; see [2]. The states and parameters are usually estimated separately and the identifier estimates are fed to the observer for state estimation. The error difference between the actual output and its estimate, which is a function of the unknown states and parameters, is made asymptotically stable in the sense of Liapunov. This approach can give unstable results when simulated on a digital computer because the numerical integration of continuous differential equations is only an approximation; see [3].

·The third kind of observer/identifier utilizes what is known as recursive least-squares algorithm. The algorithm is usually used as an identifier. An algorithm such as a Kalman filter or a stochastic approximation can be used for state observation. Both the identifier and the observer work in a bootstrap manner [4]. That is, the observer takes the identifier parameter estimates to estimate the states and, in turn, the state estimates are used by the identifier for parameter estimation.

Like most methods, these basic versions assume time-invariant linear system models. The methods are then modified to accommodate time-variant systems. Linear time-variant systems are important in control design for the following reasons:

- The design of a successful observer/identifier always relies on the accuracy of the unknown system model. Many real world dynamic control systems are nonlinear and many of them can be transformed into time-variant linear systems.
- Modern control theory techniques are based on state space representation of linear system models, and time-variant linear systems are the general form of the linear systems.

■ RECURSIVE LEAST-SQUARES ALGORITHM

When a system is deterministic or when a system noise is not available, a recursive least-squares approach may be preferred to other algorithms. Formulas for the least-squares method are derived based on the minimization of the least-squares error difference between the system output and its estimate at several time steps. Usually, the objective in the use of this algorithm is to estimate unknown constant quantities. To be specific, it is generally used to identify the system contact parameters from available input-output data. Recursive least-squares can also be used for system identification when the parameters are time-variant [5]. The parameters estimates can be improved upon if the recursive weighted least-squares algorithm is used instead.

A version of the algorithm, following [6] is given. Consider the system of equation (1) and let *F(k), G(k), and r(k) be time-invariant. Assuming the system is observable, and then according to [7]* it can be transformed to the second observable canonical form

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = c\,x_1(k) \end{cases} \quad (2)$$

where

$$A = \begin{bmatrix} 0 & & \mathbf{I} & \\ \vdots & & & \\ \mathbf{a_1} & \mathbf{a_2} & \cdots & \mathbf{a_n} \end{bmatrix}, \qquad B = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}, \qquad and \qquad c = [1\ 0 \ldots\ 0].$$

Because of the canonical structure of *A and c, the output of the system may be written as*

$$y(k+n) = v^T(k+1-n)\Theta \quad (3)$$

where

$$v^T(k+n-1) = [x_1(k), .., x_n(k), \mathrm{u(k+n+1)}, \ldots, \mathrm{u(k)}],$$

and

$$\Theta^T = [a_1, \ldots, a_n, b_1, \ldots, b_n].$$

Recursive weighted least-squares algorithm can now be applied to equation (3) for parameter estimation. Let$k = k + n - 1$, the algorithm for exponentially weighted least-squares can be described as

$$\begin{cases} \hat{\Theta}(k+1) = \hat{\Theta}(k) - K(k+1)[y(k+1) - v^T\hat{\Theta}(k)] \\ K(k+1) = \frac{P(k)v(k)}{v^T(k)P(k)v(k)+\gamma} \\ P(k+1) = [I - K(k+1)v^T]\,P(k)/\gamma \end{cases} \quad (4)$$

Where $\hat{\Theta}(k)$ is defined as the estimate of *Θ at the kth iteration. The matrix P(k) is symmetric of dimension 2n × 2n. γ(k) is the weight of the recursive least-square algorithm at the instant k. The recursive ordinary least-squares* *algorithm is obtained from (4) if γ(k) is set to unity. A special case arises when the weights γ(k) are set equal to constant γ, which lies between 0 and 1. This implies that the present observations are given a heavier weighting than the past observations. The weight γ can be regarded as a "forgetting factor" This method gives better estimates than the ordinary least-squares when the parameters are time-variant. As γ approaches 1, the memory becomes perfect, and all past observations are weighted equally. Also, a suitable choice of γ can often lead to a faster convergence of recursive algorithm in the presence of noise. It should be noted that the vector v above includes the system states which are not all known and an observer is needed to estimate the system. After the state estimates are obtained, the expressions for*$x_i(k)$ (i = 1, 2, , n) in the vector $v(k+n-1)$ may be replaced by $\hat{x}(k)$. If an initial states estimate is assumed, then an estimate of the parameters using equation (4) can be found. This parameters estimate can then be used by the observer to estimate the system states which are in turn used by the identifier of (4) to estimate the parameters. This method of estimation is known as the *bootstrap method [4].*

■ STATE ESTIMATION USING RECURSIVE LEAST-SQUARES

Consider the system given by

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = c\,x(k) \end{cases} \quad (5)$$

Where *A(k), B(k), u(k), and c are known. The objective is to use the Recursive Least-squares algorithm to estimate the states* $\hat{x}(k)$ of the system.

At the time instant

$k+1,\ y(k+1) = cx(k+1) = cA(k)x(k) + cB(k)u(k).$

Define $\overline{y}(k+1) = y(k+1) - cB(k)u(k)$ so that

$\overline{y}(k+1) = cA(k)x(k) = \overline{c}x(k)$

According to [1], the recursive least-squares can be applied to the output equation $\overline{y}(k+1) = \overline{c}x(k)$ for an estimation of the vector $x(k)$, i.e.,

$$\hat{x}(k|j+1) = \hat{x}(k|j) + K(j+1)[\overline{y}(k+1) - \overline{c}\hat{x}(k|j)] \tag{6}$$

Where j denotes the iteration index. Suppose now we want an estimate of the $x(k+1)$ states instead of $x(k)$ at the iteration j + 1. If both sides of Equation (6) are multiplied by *A(k) and the term B(k)u(k) is added to them, then the following equation results*

$$A(k)\hat{x}(k|j+1) + B(k)u(k) = A(k)\hat{x}(k|j) + B(k)u(k) + A(k)K(j+1)[y(k+1) - \overline{c}\hat{x}(k|j)]. \tag{7}$$

But according to equation (5)

$A(k)\hat{x}(k|j+1) + B(k)u(k) = \hat{x}(k+1|j+1)$

and

$A(k)\hat{x}(k|j) + B(k)u(k) = \hat{x}(k+1|j)$,These results in

$\hat{x}(k+1|j+1) = \hat{x}(k+1|j) + A(k)K(j+1)y(k+1) - c\hat{x}(k+1|j)$.(8)

From equations (4), the gain vector K(j + 1) is given by

$K(j+1) = \frac{P(j)c^T}{\overline{c}P(j)\overline{c}^T+1}$

From Equation (7),

$A(k)P(j+1) = A(k)\frac{P(j)c^T}{\overline{c}P(j)\overline{c}^T+1}$.

Let the iteration index j be the same as the sampling time index k, then the recursive least-squares algorithm for state estimation can be written as

$$\begin{cases} \hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K^*(k+1)[y(k+1) - c\hat{x}(k+1|k)] \\ K^*(k+1) = \frac{A(k)P(j)A^T(k)c^T}{cA(k)P(j)A^T(k)c^T+1} \\ P^*(k+1) = [I - K^*(k+1)c]A(k)P^*(k+1)A^T(k) \end{cases} \tag{9}$$

and $P^*(k) = P(k)$ is some initial symmetric definite matrix.

CONCLUSION

The most common algorithms that are utilized for the design of observers/ identifiers are highlighted with special emphasis on the recursive least-squares method. Recursive least-squares are best in the absence of noise or its effect is negligible.

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